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Research article

Global attractivity and almost periodic solution of a multispecies mutualism system with time-varying delays and impulsive effects

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Abstract

In this paper, we discuss an almost periodic multispecies Lotka-Volterra mutualism system with time-varying delays and impulsive effects. By using the theory of comparison theorem and constructing a suitable Lyapunov functional, sufficient conditions which guarantee the existence and uniqueness and global attractivity of almost periodic solution of the system are obtained. An suitable example is employed to illustrate the feasibility of the main results.

Keywords: Almost periodic solution; Mutualism system; Time-varying delay; Impulsive effect; Global attractivity

1 Introduction

In real world phenomenon, the environment varies due to the factors such as seasonal effects of weather, food supplies, mating habits, harvesting. So it is usual to assume the periodicity of parameters in the systems. However, if the various constituent components of the temporally nonuniform environment is with incommensurable (nonintegral multiples) periods, then one has to consider the environment to be almost periodic since there is no a priori reason to expect the existence of periodic solutions. For this reason, the assumption of almost periodicity is more realistic, more important and more general when we consider the effects of the environmental factors. Recently, there have been many nice works on the positive almost periodic solutions of continuous and discrete dynamics model with almost periodic coefficients(see [1–11] and the references cited therein).

In this paper, we are concerned with the following multispecies Lotka-Volterra mutualism system with time-varying delays and impulsive effects

$$\begin{cases} \dot{x}_i(t) = x_i(t)\left[a_i(t) - b_i(t)x_i(t-\tau_i(t)) + \displaystyle\sum_{j=1,j\neq i}^{n} c_{ij}(t)\frac{x_j(t-\sigma_{ij}(t))}{1+x_j(t-\sigma_{ij}(t))}\right], \quad t\neq t_k, \\ x_i(t_k^+) = (1+h_{ik})x_i(t_k), \quad k\in \mathbf{Z}^+, \quad i=1,2,\cdots,n, \end{cases} \tag{1.1}$$

with initial conditions

$$x_i(\theta)=\phi_i(\theta), \quad \theta\in[-\tau,0], \quad \phi_i(\theta)\in C([-\tau,0], \quad \mathbf{R}^+), \quad i=1,2,\cdots,n, \tag{1.2}$$

where $x_i(t)$ are the ith species population density at time t, $a_i(t)$ represent the population grow rate of the species x_i; $b_i(t)$ and $\tau_i(t)$ represent the population decay rate and time delays in the competition among the ith species, respectively; $c_{ij}(t)$ and $\sigma_{ij}(t)$ represent the ith species population increase rate and time delays in the mutualism among the other species $x_j (i,j=1,2,\cdots,n, i\neq j)$; the constant τ is

$$\tau = \max\big\{\max_{1\le i\le n}\big\{\sup_{t\in\mathbf{R}^+}\tau_i(t)\big\}, \max_{1\le i,j\le n, j\neq i}\big\{\sup_{t\in\mathbf{R}^+}\sigma_{ij}(t)\big\}\big\};$$

$h_{ik}>-1$, $i=1,2,\cdots,n, k\in\mathbf{Z}^+$ are constants and $0=t_0<t_1<t_2<\cdots<t_k<t_{k+1}<\cdots$, are impulse points with $\lim\limits_{k\to+\infty} t_k=+\infty$.

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For any bounded function $f(t)$ defined on $\mathbf{R}$, define

$$f^u = \sup_{t\in\mathbf{R}} f(t), \quad f^l = \inf_{t\in\mathbf{R}} f(t).$$

Throughout this paper, we assume that

(H1) $a_i(t), b_i(t)$ and $c_{ij}(t)$ are all bounded continuous almost periodic functions such that $a_i^l > 0$, $b_i^l > 0$ and $c_{ij}^l > 0$, $i, j = 1, 2, \cdots, n, j \neq i$.

(H2) $H_i(t) = \prod\limits_{0<t_k<t} (1 + h_{ik}), i = 1, 2, \cdots, n, k \in \mathbf{Z}^+$ are bounded almost periodic functions and there exists positive constants H_i^u and H_i^l such that $H_i^l \leq H_i(t) \leq H_i^u$.

(H3) $\tau_i(t)$ and $\sigma_{ij}(t)$ are positive and continuously differentiable almost periodic functions on $\mathbf{R}^+$ such that $\tau_i(0) = 0$, $\sigma_{ij}(0) = 0$, $\dot{\tau}_i(t) < 1$ and $\dot{\sigma}_{ij}(t) < 1$, which imply that the function $\varphi_i(t) = t-\tau_i(t)$, $\psi_{ij}(t) = t-\sigma_{ij}(t)$ exist the inverse functions $\varphi_i^{-1}(t)$, $\psi_{ij}^{-1}(t)$ and $\varphi_i^{-1}(t) > \varphi_i(t)$, $\psi_{ij}^{-1}(t) > \psi_{ij}(t)$ for $t \geq 0$.

To the best of our knowledge, this are few papers to investigate the global attractivity of positive almost periodic solution of multispecies Lotka-Volterra mutualism system with time-varying delays and impulsive effects. The aim of this paper is to obtain sufficient conditions for the existence of a unique globally attractive almost periodic solution of the systems (1.1) and (1.2), by utilizing the comparison theorem of the differential equation and constructing a suitable Lyapunov functional and applying the analysis technique of papers [1, 12–18].

The remaining part of this paper is organized as follows: In Section 2, we will introduce some definitions and several useful lemmas. In Section 3, by applying the theory of differential inequality, we present the permanence results for systems (1.1) and (1.2). In Section 4, by constructing a suitable Lyapunov function, we establish the sufficient conditions which ensure the global attractivity of the system (2.1). In Section 5, some sufficient conditions which guarantee existence and uniqueness of almost periodic solution of the systems (1.1) and (1.2) are obtained. A suitable example is given to illustrate the feasibility of the main results in Section 6. Finally, the conclusion ends with brief remarks.

2 Preliminaries

Firstly, we give the definitions of the terminologies involved.

Definition 2.1([19]) A function $f(t, x)$, where f is an $m-$vector, t is a real scalar and x is an $n-$vector, is said to be almost periodic in t uniformly with respect to $x \in X \subset \mathbf{R}^n$, if $f(t, x)$ is continuous in $t \in \mathbf{R}$ and $x \in X$, and if for any $\varepsilon > 0$, it is possible to find a constant $l(\varepsilon) > 0$ such that in any interval of length $l(\varepsilon)$ there exists a τ such that the inequality

$$\| f(t+\tau, x) - f(t, x) \| = \sum_{i=1}^{m} |f_i(t+\tau, x) - f_i(t, x)| < \varepsilon$$

is satisfied for all $t \in \mathbf{R}, x \in X$. The number τ is called an $\varepsilon-$translation number of $f(t, x)$.

Definition 2.2([16]) A function $f : \mathbf{R} \to \mathbf{R}$ is said to be asymptotically almost periodic function if there exists an almost periodic function $q(t)$ and a continuous function $r(t)$ such that

$$f(t) = q(t) + r(t), t \in R \ \text{ and } \ r(t) \to 0 \ \text{ as } \ t \to \infty.$$

For the relevant definitions and the properties of almost periodic functions, we refer to [20, 21].

Definition 2.3 If $(x_1(t), x_2(t), \cdots, x_n(t))^T$ is a positive solution of systems (1.1) and (1.2), $(\bar{x}_1(t), \bar{x}_2(t), \cdots, \bar{x}_n(t))^T$ is any positive solution of systems (1.1) and (1.2) satisfying

$$\lim_{t\to+\infty} \sum_{i=1}^{n} |\bar{x}_i(t) - x_i(t)| = 0,$$

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then we say $(x_1(t), x_2(t), \cdots, x_n(t))^T$ is globally asymptotically stable.

From the point of view of biology, in the sequel, we assume that $\mathbf{x}(t_0) = (x_1(t_0), x_2(t_0), \cdots, x_n(t_0))^T > \mathbf{0}$ for some $t_0 \geq 0$. Then it is easy to see that, for given $\mathbf{x}(t_0) > \mathbf{0}$, the systems (1.1) and (1.2) have a positive solution $\mathbf{x}(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ passing through $\mathbf{x}(t_0)$ for $t \in \mathbf{R}^+$.

Lemma 2.1 $\{(x_1(t), x_2(t), \cdots, x_n(t))^T \in \mathbf{R}^{+n} | x_i(t_0) > 0 \text{ for } t_0 \geq 0, i = 1, 2, \cdots, n\}$ is positive invariant with respect to the systems (1.1) and (1.2).

Proof. For $x_i(t_0) > 0 (i = 1, 2, \cdots, n)$, then we get

$$x_i(t) = x_i(t_0) \exp\left\{ \int_{t_0}^{t} \left[a_i(s) - b_i(s) x_i(s - \tau_i(s)) + \sum_{j=1, j \neq i}^{n} c_{ij}(s) \frac{x_j(s - \sigma_{ij}(s))}{1 + x_j(s - \sigma_{ij}(s))} \right] ds \right\} > 0.$$

Thus, we prove Lemma 2.1.

Lemma 2.2([22]) If $\dot{x} \geq (\leq) x(b - a x^c)$, where a, b, c are positive constant, then

$$\liminf_{t \to +\infty} x(t) \geq (\frac{b}{a})^{\frac{1}{c}} \left(\limsup_{t \to +\infty} x(t) \leq (\frac{b}{a})^{\frac{1}{c}} \right).$$

Lemma 2.3([23]) Suppose that the continuous operator A maps the the closed and bounded convex set $Q \subset \mathbf{R}^n$ onto itself, then the operator A has at least one fixed point in set Q.

Consider the following system

$$\dot{y}_i(t) = y_i(t) \left[a_i(t) - B_i(t) y_i(t - \tau_i(t)) + \sum_{j=1, j \neq i}^{n} C_{ij}(t) \frac{y_j(t - \sigma_{ij}(t))}{1 + H_j(t) y_j(t - \sigma_{ij}(t))} \right], \quad i = 1, 2, \cdots, n, \tag{2.1}$$

with initial value $y_i(s) = \phi_i(s), s \in [-\tau, 0]$, ϕ is defined as that in (1.2), and

$$B_i(t) = \prod_{0 < t_k < t} (1 + h_{ik}) b_i(t), \quad C_{ij}(t) = \prod_{0 < t_k < t} (1 + h_{jk}) c_{ij}(t), \quad j \neq i.$$

Lemma 2.4 $\{(y_1(t), y_2(t), \cdots, y_n(t))^T \in \mathbf{R}^{+n} | y_i(t_0) > 0 \text{ for } t_0 \geq 0, i = 1, 2, \cdots, n\}$ is positive invariant with respect to the system (2.1).

Lemma 2.5 For system (1.1) and (2.1), the following results hold:

(1) if $(y_1(t), y_2(t), \cdots, y_n(t))^T$ is a solution of system (2.1), then

$$(x_1(t), x_2(t), \cdots, x_n(t))^T = \Big(\prod_{0 < t_k < t} (1 + h_{1k}) y_1(t), \prod_{0 < t_k < t} (1 + h_{2k}) y_2(t), \cdots, \prod_{0 < t_k < t} (1 + h_{nk}) y_n(t) \Big)^T$$

is a solution of system (1.1);

(2) if $(x_1(t), x_2(t), \cdots, x_n(t))^T$ is a solution of system (1.1), then

$$(y_1(t), y_2(t), \cdots, y_n(t))^T = \Big(\prod_{0 < t_k < t} (1 + h_{1k})^{-1} x_1(t), \prod_{0 < t_k < t} (1 + h_{2k})^{-1} x_2(t), \cdots, \prod_{0 < t_k < t} (1 + h_{nk})^{-1} x_n(t) \Big)^T$$

is a solution of system (2.1).

Proof. The proof of Lemma 2.5 is similar to the proof of Lemma 2.5 in [1] and we omit the details here.

3 Permanence

In this section, we establish permanence results for systems (1.1) and (1.2), which can be given by Lemma 2.2. The proofs of following results are similar to the proofs in [1] and we omit the details here.

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Theorem 3.1 Assume that (H1)-(H3) hold. Then any positive solution $(x_1(t), x_2(t), \cdots, x_n(t))^T$ of systems (1.1) and (1.2) satisfies

$$m_i \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le M_i, \qquad i = 1, 2, \cdots, n,$$

where

$$M_i = \frac{a_i^u + \sum\limits_{j=1,j\neq i}^{n} c_{ij}^u}{b_i^l} \exp\left\{\left(a_i^u + \sum_{j=1,j\neq i}^{n} c_{ij}^u\right)\tau\right\}, \quad m_i = \frac{a_i^l}{b_i^u}\exp\{(a_i^l - b_i^u M_i)\tau\}.$$

That is, systems (1.1) and (1.2) is permanent.

Theorem 3.2 Assume that (H1)-(H3) hold. Then any positive solution $(y_1(t), y_2(t), \cdots, y_n(t))^T$ of system (2.1) satisfies

$$\frac{m_i}{H_i^u} \le \liminf_{t\to+\infty} y_i(t) \le \limsup_{t\to+\infty} x_i(t) \le \frac{M_i}{H_i^l}, \qquad i = 1, 2, \cdots, n.$$

That is, system (2.1) is permanent.

The next result tells us that there exist positive solutions of system (2.1) for $t \in \mathbf{R}^+$ totally in the interval of Theorem 3.2. To be precise:

Theorem 3.3 System (2.1) has at least one positive solution $(y_1(t), y_2(t), \cdots, y_n(t))^T$ satisfying

$$\frac{m_i}{H_i^u} \le y_i(t) \le \frac{M_i}{H_i^l} \quad \text{for } t \in \mathbf{R}^+.$$

4 Global attractivity

In this section, we establish the global asymptotical stability of system (2.1).

Theorem 4.1 Assume that the system (2.1) satisfy condition (H1)-(H3) and the following conditions:

$$\text{(H4)} \quad \liminf_{t\to+\infty} G_i(t) > 0, \quad i = 1, 2, \cdots, n,$$

where

$$\begin{aligned} G_i(t) \;=\; & B_i(t) - \left[a_i(t) + \frac{M_i}{H_i^l}B_i(t) + \sum_{j=1,j\neq i}^{n} \frac{M_j}{H_j^l(1+m_j)}C_{ij}(t)\right]\int_t^{\varphi_i^{-1}(t)} B_i(u)du \\ & - \frac{M_i B_i(\varphi_i^{-1}(t))}{H_i^l \dot{\varphi}_i(\varphi_i^{-1}(t))}\int_{\varphi_i^{-1}(t)}^{\varphi_i^{-1}(\varphi_i^{-1}(t))} B_i(u)du \\ & - \frac{1}{(1+m_i)^2}\sum_{j=1,j\neq i}^{n} \frac{C_{ji}(\psi_{ji}^{-1}(t))}{\dot{\psi}_{ji}(\psi_{ji}^{-1}(t))}\left[1 + \frac{M_j}{H_j^l}\int_{\psi_{ji}^{-1}(t)}^{\varphi_j^{-1}(\psi_{ji}^{-1}(t))} B_j(u)du\right], \end{aligned}$$

in which φ_i^{-1} and ψ_{ij}^{-1} are the inverse function of $\varphi_i(t) = t - \tau_i(t)$ and $\psi_{ij}(t) = t - \sigma_{ij}(t)$, $i, j = 1, 2, \cdots, n,\ j \neq i$, respectively.

Then the solution of system (2.1) is globally attractive.

Proof. Let $y(t) = (y_1(t), y_2(t), \cdots, y_n(t))^T$ and $\bar{y}(t) = (\bar{y}_1(t), \bar{y}_2(t), \cdots, \bar{y}_n(t))^T$ be any two positive solutions of the system (2.1).

From Theorem 3.2, there exists a positive constant $T > 0$, such that

$$\frac{m_i}{H_i^u} \le y_i(t) \le \frac{M_i}{H_i^l}, \quad \text{for} \quad t > T, \qquad i = 1, 2, \cdots, n.$$

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Let

$$V_{i1}(t) = |\ln \bar{y}_i(t) - \ln y_i(t)|, \quad i = 1, 2, \cdots, n,$$

then we get the upper right derivative of V_{i1} along system (2.1)

$$\begin{aligned}
&D^+V_{i1}(t) = sign(\bar{y}_i(t) - y_i(t))\Big(\frac{\dot{\bar{y}}_i(t)}{\bar{y}_i(t)} - \frac{\dot{y}_i(t)}{y_i(t)}\Big)\\
&= sign(\bar{y}_i(t) - y_i(t))\Big\{ - B_i(t)\big[\bar{y}_i(t-\tau_i(t)) - y_i(t-\tau_i(t))\big]\\
&\quad + \sum_{j=1,j\neq i}^{n} C_{ij}(t)\big[\frac{\bar{y}_j(t-\sigma_{ij}(t))}{1+H_j(t)\bar{y}_j(t-\sigma_{ij}(t))} - \frac{y_j(t-\sigma_{ij}(t))}{1+H_j(t)y_j(t-\sigma_{ij}(t))}\big]\Big\}\\
&\leq sign(\bar{y}_i(t) - y_i(t))\Big\{ - B_i(t)\big[\bar{y}_i(t-\tau_i(t)) - y_i(t-\tau_i(t))\big]\Big\}\\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2}\big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big|.
\end{aligned}$$

By using inequality $-sign(a)\cdot b \leq -|a| + |a-b| (a, b \in \mathbf{R})$ we obtain that

$$\begin{aligned}
D^+V_{i1}(t) &\leq -B_i(t)|\bar{y}_i(t) - y_i(t)| + B_i(t)\Big|\int_{t-\tau_i(t)}^{t}(\dot{\bar{y}}_i(s) - \dot{y}_i(s))ds\Big|\\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2}\big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big|.
\end{aligned} \tag{4.1}$$

On substituting the equations of system (2.1) into (4.1) yields

$$\begin{aligned}
&D^+V_{i1}(t) \leq -B_i(t)|\bar{y}_i(t) - y_i(t)|\\
&\quad + B_i(t)\Big|\int_{t-\tau_i(t)}^{t}\Big\{\bar{y}_i(s)\Big[a_i(s) - B_i(s)\bar{y}_i(s-\tau_i(s)) + \sum_{j=1,j\neq i}^{n} C_{ij}(s)\frac{\bar{y}_j(s-\sigma_{ij}(s))}{1+H_j(s)\bar{y}_j(s-\sigma_{ij}(s))}\Big]\\
&\quad - y_i(s)\Big[a_i(s) - B_i(s)y_i(s-\tau_i(s)) + \sum_{j=1,j\neq i}^{n} C_{ij}(s)\frac{y_j(s-\sigma_{ij}(s))}{1+H_j(t)y_j(s-\sigma_{ij}(s))}\Big]\Big\}ds\Big|\\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2}\big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big|\\
&= -B_i(t)|\bar{y}_i(t) - y_i(t)|\\
&\quad + B_i(t)\Big|\int_{t-\tau_i(t)}^{t}\Big\{\Big[a_i(s) - B_i(s)\bar{y}_i(s-\tau_i(s)) + \sum_{j=1,j\neq i}^{n} C_{ij}(s)\frac{\bar{y}_j(s-\sigma_{ij}(s))}{1+H_j(s)\bar{y}_j(s-\sigma_{ij}(s))}\Big](\bar{y}_i(s) - y_i(s))\\
&\quad - B_i(s)y_i(s)[\bar{y}_i(s-\tau_i(s)) - y_i(s-\tau_i(s))]\\
&\quad - y_i(s)\Big[\sum_{j=1,j\neq i}^{n} C_{ij}(s)\frac{y_j(s-\sigma_{ij}(s))}{1+H_j(s)y_j(s-\sigma_{ij}(s))} - \sum_{j=1,j\neq i}^{n} C_{ij}(s)\frac{\bar{y}_j(s-\sigma_{ij}(s))}{1+H_j(s)\bar{y}_j(s-\sigma_{ij}(s))}\Big]\Big\}ds\Big|\\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2}\big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big|.
\end{aligned} \tag{4.2}$$

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It follows from (4.2) that for $t \geq T + \tau$

$$
\begin{aligned}
D^+V_{i1}(t) &\leq -B_i(t)|\bar{y}_i(t) - y_i(t)| \\
&\quad + B_i(t) \int_{t-\tau_i(t)}^{t} \Bigg\{ \Bigg[a_i(s) + B_i(s)\bar{y}_i(s-\tau_i(s)) + \sum_{j=1, j\neq i}^{n} C_{ij}(s) \frac{\bar{y}_j(s-\sigma_{ij}(s))}{1 + H_j(s)\bar{y}_j(s-\sigma_{ij}(s))} \Bigg] |\bar{y}_i(s) - y_i(s)| \\
&\quad + B_i(s)y_i(s)|\bar{y}_i(s-\tau_i(s)) - y_i(s-\tau_i(s))| \\
&\quad + y_i(s) \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(s)}{(1+m_j)^2} |\bar{y}_j(s-\sigma_{ij}(s)) - y_j(s-\sigma_{ij}(s))| \Bigg\} ds \\
&\quad + \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2} \big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big| \\
&\leq -B_i(t)|\bar{y}_i(t) - y_i(t)| \\
&\quad + B_i(t) \int_{t-\tau_i(t)}^{t} \Bigg\{ \Bigg[a_i(s) + \frac{M_i}{H_i^l} B_i(s) + \sum_{j=1, j\neq i}^{n} \frac{M_j}{H_j^l(1+m_j)} C_{ij}(s) \Bigg] |\bar{y}_i(s) - y_i(s)| \\
&\quad + \frac{M_i}{H_i^l} B_i(s)|\bar{y}_i(s-\tau_i(s)) - y_i(s-\tau_i(s))| \\
&\quad + \frac{M_i}{H_i^l} \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(s)}{(1+m_j)^2} |\bar{y}_j(s-\sigma_{ij}(s)) - y_j(s-\sigma_{ij}(s))| \Bigg\} ds \\
&\quad + \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2} \big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big| \\
&= -B_i(t)|\bar{y}_i(t) - y_i(t)| + B_i(t) \int_{t-\tau_i(t)}^{t} F_i(s) ds + \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2} \big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big| \\
&= -B_i(t)|\bar{y}_i(t) - y_i(t)| + B_i(t)[P_i(t) - P_i(\varphi_i(t))] \\
&\quad + \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2} \big|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))\big|, \qquad (4.3)
\end{aligned}
$$

where

$$
\begin{aligned}
F_i(s) &= \Bigg[a_i(s) + \frac{M_i}{H_i^l} B_i(s) + \sum_{j=1, j\neq i}^{n} \frac{M_j}{H_j^l(1+m_j)} C_{ij}(s) \Bigg] |\bar{y}_i(s) - y_i(s)| \\
&\quad + \frac{M_i}{H_i^l} B_i(s)|\bar{y}_i(s-\tau_i(s)) - y_i(s-\tau_i(s))| + \frac{M_i}{H_i^l} \sum_{j=1, j\neq i}^{n} \frac{C_{ij}(s)}{(1+m_j)^2} |\bar{y}_j(s-\sigma_{ij}(s)) - y_j(s-\sigma_{ij}(s))|,
\end{aligned}
$$

and $P_i(s)$ is a primitive function of $F_i(s)$, $i, j = 1, 2, \cdots, n,\ j \neq i$.

Define

$$
V_{i2}(t) = \int_{t}^{\varphi_i^{-1}(t)} \int_{\varphi_i(u)}^{t} B_i(u) F_i(s) ds du + \sum_{j=1, j\neq i}^{n} \frac{1}{(1+m_j)^2} \int_{\psi_{ij}(t)}^{t} \frac{C_{ij}(\psi_{ij}^{-1}(u))}{\dot{\psi}_{ij}(\psi_{ij}^{-1}(u))} |\bar{y}_j(u) - y_j(u)| du, \qquad (4.4)
$$

we can easily get that

$$
\begin{aligned}
V_{i2}(t) &= \int_{t}^{\varphi_i^{-1}(t)} B_i(u)[P_i(t) - P_i(\varphi_i(u))] du + \sum_{j=1, j\neq i}^{n} \frac{1}{(1+m_j)^2} \int_{\psi_{ij}(t)}^{t} \frac{C_{ij}(\psi_{ij}^{-1}(u))}{\dot{\psi}_{ij}(\psi_{ij}^{-1}(u))} |\bar{y}_j(u) - y_j(u)| du \\
&= P_i(t) \int_{t}^{\varphi_i^{-1}(t)} B_i(u) du - \int_{t}^{\varphi_i^{-1}(t)} B_i(u) P_i(\varphi_i(u)) du \\
&\quad + \sum_{j=1, j\neq i}^{n} \frac{1}{(1+m_j)^2} \int_{\psi_{ij}(t)}^{t} \frac{C_{ij}(\psi_{ij}^{-1}(u))}{\dot{\psi}_{ij}(\psi_{ij}^{-1}(u))} |\bar{y}_j(u) - y_j(u)| du.
\end{aligned}
$$

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Then, we obtain that for $t \geq T+\tau$

$$\begin{aligned} D^+V_{i2}(t) &= F_i(t)\int_t^{\varphi_i^{-1}(t)} B_i(u)du + P_i(t)[\frac{1}{\dot{\varphi}_i(t)}B_i(\varphi_i^{-1}(t)) - B_i(t)] \\ &\quad -[\frac{1}{\dot{\varphi}_i(t)}B_i(\varphi_i^{-1}(t))P_i(\varphi_i(\varphi_i^{-1}(t))) - B_i(t)P_i(\varphi_i(t))] \\ &\quad + \sum_{j=1,j\neq i}^{n} \frac{1}{(1+m_j)^2}\frac{C_{ij}(\psi_{ij}^{-1}(t))}{\dot{\psi}_{ij}(\psi_{ij}^{-1}(t))}|\bar{y}_j(t) - y_j(t)| \\ &\quad - \sum_{j=1,j\neq i}^{n} \frac{1}{(1+m_j)^2}\dot{\psi}_{ij}(t)\frac{C_{ij}(\psi_{ij}^{-1}(\psi_{ij}(t)))}{\dot{\psi}_{ij}(\psi_{ij}^{-1}(\psi_{ij}(t)))}|\bar{y}_j(\psi_{ij}(t)) - y_j(\psi_{ij}(t))| \\ &= F_i(t)\int_t^{\varphi_i^{-1}(t)} B_i(u)du - B_i(t)[P_i(t) - P_i(\varphi_i(t))] \\ &\quad + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(\psi_{ij}^{-1}(t))}{(1+m_j)^2\dot{\psi}_{ij}(\psi_j^{-1}(t))}|\bar{y}_j(t) - y_j(t)| \\ &\quad - \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t)}{(1+m_j)^2}|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))|. \end{aligned} \tag{4.5}$$

Define

$$\begin{aligned} V_{i3}(t) &= \frac{M_i}{H_i^l}\int_{\varphi_i(t)}^{t}\int_{\varphi_i^{-1}(l)}^{\varphi_i^{-1}(\varphi_i^{-1}(l))} \frac{B_i(u)B_i(\varphi_i^{-1}(l))}{\dot{\varphi}_i(\varphi_i^{-1}(l))}|\bar{y}_i(l) - y_i(l)|dudl \\ &\quad + \frac{M_i}{H_i^l}\sum_{j=1,j\neq i}^{n}\int_{\psi_{ij}(t)}^{t}\int_{\psi_{ij}^{-1}(l)}^{\varphi_i^{-1}(\psi_{ij}^{-1}(l))} \frac{B_i(u)C_{ij}(\psi_{ij}^{-1}(l))}{(1+m_j)^2\dot{\psi}_{ij}(\psi_{ij}^{-1}(l))}|\bar{y}_j(l) - y_j(l)|dudl, \end{aligned} \tag{4.6}$$

we obtain that for $t \geq T+\tau$

$$\begin{aligned} D^+V_{i3}(t) &= \frac{M_iB_i(\varphi_i^{-1}(t))}{H_i^l\dot{\varphi}_i(\varphi_i^{-1}(t))}\int_{\varphi_i^{-1}(t)}^{\varphi_i^{-1}(\varphi_i^{-1}(t))} B_i(u)du|\bar{y}_i(t) - y_i(t)| \\ &\quad - \frac{M_i}{H_i^l}B_i(t)|\bar{y}_i(t-\tau_i(t)) - y_i(t-\tau_i(t))|\int_t^{\varphi_i^{-1}(t)} B_i(u)du \\ &\quad + \frac{M_i}{H_i^l}\sum_{j=1,j\neq i}^{n}\frac{C_{ij}(\psi_{ij}^{-1}(t))}{(1+m_j)^2\dot{\psi}_{ij}(\psi_{ij}^{-1}(t))}\int_{\psi_{ij}^{-1}(t)}^{\varphi_i^{-1}(\psi_{ij}^{-1}(t))} B_i(u)du|\bar{y}_j(t) - y_j(t)| \\ &\quad - \frac{M_i}{H_i^l}\sum_{j=1,j\neq i}^{n}\frac{C_{ij}(t)}{(1+m_j)^2}\int_t^{\varphi_i^{-1}(t)} B_i(u)du|\bar{y}_j(t-\sigma_{ij}(t)) - y_j(t-\sigma_{ij}(t))|. \end{aligned} \tag{4.7}$$

Define

$$V_i(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t), \quad i = 1, 2, \cdots, n,$$

it then follows from (4.3)-(4.7) that for $t \geq T+\tau$

$$\begin{aligned} D^+V_i(t) &\leq -B_i(t)|\bar{y}_i(t) - y_i(t)| \\ &\quad + \Big[a_i(t) + \frac{M_i}{H_i^l}B_i(t) + \sum_{j=1,j\neq i}^{n}\frac{M_j}{H_j^l(1+m_j)}C_{ij}(t)\Big]\int_t^{\varphi_i^{-1}(t)} B_i(u)du|\bar{y}_i(t) - y_i(t)| \\ &\quad + \sum_{j=1,j\neq i}^{n}\frac{C_{ij}(\psi_{ij}^{-1}(t))}{(1+m_j)^2\dot{\psi}_{ij}(\psi_{ij}^{-1}(t))}|\bar{y}_j(t) - y_j(t)| \\ &\quad + \frac{M_iB_i(\varphi_i^{-1}(t))}{H_i^l\dot{\varphi}_i(\varphi_i^{-1}(t))}\int_{\varphi_i^{-1}(t)}^{\varphi_i^{-1}(\varphi_i^{-1}(t))} B_i(u)du|\bar{y}_i(t) - y_i(t)| \\ &\quad + \frac{M_i}{H_i^l}\sum_{j=1,j\neq i}^{n}\frac{C_{ij}(\psi_{ij}^{-1}(t))}{(1+m_j)^2\dot{\psi}_{ij}(\psi_{ij}^{-1}(t))}\int_{\psi_{ij}^{-1}(t)}^{\varphi_i^{-1}(\psi_{ij}^{-1}(t))} B_i(u)du|\bar{y}_j(t) - y_j(t)|. \end{aligned} \tag{4.8}$$

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Define Lyapunov function $V(t)$ as

$$V(t) = \sum_{i=1}^{n} V_i(t),$$

it follows from (4.8) that for $t \geq T + \tau$

$$D^+V(t) \leq -\sum_{i=1}^{n} G_i(t)|\bar{y}_i(t) - y_i(t)|, \tag{4.9}$$

where $G_i(t)$ is defined in Theorem 4.1.

By condition (H4), there exist positive constants $\beta_i (i = 1, 2, \cdots, n)$ and $T_0 \geq T + \tau$ such that if $t > T_0$

$$G_i(t) \geq \beta_i > 0. \tag{4.10}$$

Let $\beta^* = \min\{\beta_1, \beta_2, \cdots, \beta_n\}$, we obtain from (4.9) and (4.10)

$$D^+V(t) \leq -\beta^* \sum_{i=1}^{n} |\bar{y}_i(t) - y_i(t)|, \tag{4.11}$$

Integrating both sides of (4.11) on interval $[T_0, t]$, we obtain

$$V(t) + \beta^* \int_{T_0}^{t} \sum_{i=1}^{n} |\bar{y}_i(u) - y_i(u)| du \leq V(T_0), \quad \text{for} \quad t \geq T_0.$$

Therefore, $V(t)$ is bounded on $[T_0, +\infty)$ and also

$$\int_{T_0}^{+\infty} \sum_{i=1}^{n} |\bar{y}_i(u) - y_i(u)| du \leq +\infty.$$

From system (2.1) and Theorem 3.2, we can obtain that

$$\bar{y}_i(t) - y_i(t), \quad i = 1, 2, \cdots, n$$

and their derivatives remain bounded on $[T_0, +\infty)$. Therefore

$$\sum_{i=1}^{n} |\bar{y}_i(t) - y_i(t)|$$

is uniformly continuous. By Barbalat Lemma([24]), we conclude that

$$\lim_{t \to +\infty} \sum_{i=1}^{n} |\bar{y}_i(t) - y_i(t)| = 0$$

and here

$$\lim_{t \to +\infty} |\bar{y}_i(t) - y_i(t)| = 0, \quad i = 1, 2, \cdots, n. \tag{4.12}$$

Then the positive solution of system (2.1) is globally attractive. This completes this proof of Theorem 4.1.

5 Almost periodic solution

The main result of this paper concerns the existence and uniqueness of positive almost periodic solution of systems (1.1) and (1.2) which is globally attractive. We first prove that system (2.1) has a unique globally attractive positive almost periodic solution. To be precise:

Theorem 5.1 Assume that (H1)-(H4) hold. Then system (2.1) admits a unique positive almost periodic solution which is globally attractive.

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Proof. From Theorem 3.3, there exists a bounded positive solution $(y_1(t), y_2(t), \cdots, y_n(t))^T$ of system (2.1) satisfying

$$\frac{m_i}{H_i^u} \leq y_i(t) \leq \frac{M_i}{H_i^l}, \quad t \in \mathbf{R}^+.$$

Then there exists a sequence $\{\delta_p'\}$ with $\delta_p' \to \infty$ as $p \to \infty$ such that $(y_1(t+\delta_p'), y_2(t+\delta_p'), \cdots, y_n(t+\delta_p'))^T$ is a solution of the following system

$$\dot{y}_i(t) = y_i(t)\left[a_i(t+\delta_p') - B_i(t+\delta_p')y_i(t-\tau_i(t)) + \sum_{j=1,j\neq i}^{n} \frac{C_{ij}(t+\delta_p')y_j(t-\sigma_{ij}(t))}{1+H_j(t+\delta_p')y_j(t-\sigma_{ij}(t))}\right], \; i = 1, 2, \cdots, n. \quad (5.1)$$

From above and Theorem 3.1, we have that not only $(y_1(t+\delta_p'), y_2(t+\delta_p'), \cdots, y_n(t+\delta_p'))^T$ but also $(\dot{y}_1(t+\delta_p'), \dot{y}_2(t+\delta_p'), \cdots, \dot{y}_n(t+\delta_p'))^T$ are uniformly bounded, thus $(y_1(t+\delta_p'), y_2(t+\delta_p'), \cdots, y_n(t+\delta_p'))^T$ is uniformly bounded and equi-continuous. By Ascoli theorem [25], there exists a uniformly convergent subsequence $\{(y_1(t+\delta_p), y_2(t+\delta_p), \cdots, y_n(t+\delta_p))^T\} \subseteq \{(y_1(t+\delta_p'), y_2(t+\delta_p'), \cdots, y_n(t+\delta_p'))^T\}$ such that for any $\forall\varepsilon > 0$, there exists a $\lambda_0(\varepsilon) > 0$ with the property that if $p, q > \lambda_0(\varepsilon)$ then

$$|y_i(t+\delta_p) - y_i(t+\delta_q)| < \varepsilon, \quad i = 1, 2, \cdots, n,$$

which shows that $(y_1(t), y_2(t), \cdots, y_n(t))$ is asymptotically almost periodic function of system (2.1). So, by Definition 2.2, there exist $p_i(t)$ and $q_i(t)$, for $i = 1, 2, \cdots, n, t \in \mathbf{R}^+$, such that

$$y_i(t) = p_i(t) + q_i(t), \quad t \in \mathbf{R}^+,$$

where

$$\lim_{p\to+\infty} p_i(t+\delta_p) = p_i(t), \quad \lim_{p\to+\infty} q_i(t+\delta_p) = 0,$$

$p_i(t)$ are almost periodic functions. It means that

$$\lim_{p\to+\infty} y_i(t+\delta_p) = p_i(t), \quad i = 1, 2, \cdots, n.$$

On the other hand,

$$\begin{aligned}
\lim_{p\to+\infty} \dot{y}_i(t+\delta_p) &= \lim_{p\to+\infty} \lim_{h\to 0} \frac{y_i(t+\delta_p+h) - y_i(t+\delta_p)}{h} \\
&= \lim_{h\to 0} \lim_{p\to+\infty} \frac{y_i(t+\delta_p+h) - y_i(t+\delta_p)}{h} = \lim_{h\to 0} \frac{p_i(t+h) - p_i(t)}{h},
\end{aligned} \quad (5.2)$$

so the function $\dot{p}_i(t)(i = 1, 2, \cdots, n)$ exist.

Now we will prove that

$$p(t) = (p_1(t), p_2(t), \cdots, p_n(t))^T$$

is an almost periodic solution of system (2.1).

From properties of almost periodic function, there exists a sequence $\{\delta_\lambda\}$, $\delta_\lambda \to \infty$ as $\lambda \to +\infty$, such that

$$a_i(t+\delta_\lambda) \to a_i(t), \quad B_i(t+\delta_\lambda) \to B_i(t), \quad C_{ij}(t+\delta_\lambda) \to C_{ij}(t), \quad H_i(t+\delta_\lambda) \to H_i(t), \quad \tau_i(t+\delta_\lambda) \to \tau_i(t),$$

as $\lambda \to +\infty$ uniformly on $\mathbf{R}^+$, $i = 1, 2, \cdots, n$.

It is not difficult to know that

$$\lim_{\lambda\to+\infty} y_i(t+\delta_\lambda) = p_i(t), \quad i = 1, 2, \cdots, n,$$

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then we have

$$\begin{aligned}\dot{p}_i(t) &= \lim_{\lambda\to+\infty}\dot{y}_i(t+\delta_\lambda)\\ &= \lim_{\lambda\to+\infty} y_i(t+\delta_\lambda)\Big[a_i(t+\delta_\lambda)-B_i(t+\delta_\lambda)y_i(t+\delta_\lambda-\tau_i(t+\delta_\lambda))\\ &\quad+\sum_{j=1,j\neq i}^{n} C_{ij}(t+\delta_\lambda)\frac{y_j(t+\delta_\lambda-\sigma_{ij}(t+\delta_\lambda))}{1+H_j(t+\delta_\lambda)y_j(t+\delta_\lambda-\sigma_{ij}(t+\delta_\lambda))}\Big]\\ &= p_i(t)\Big[a_i(t)-B_i(t)y_i(t-\tau_i(t))+\sum_{j=1,j\neq i}^{n} C_{ij}(t)\frac{y_j(t-\sigma_{ij}(t))}{1+H_j(t)y_j(t-\sigma_{ij}(t))}\Big].\end{aligned}$$

This prove that $p(t)=(p_1(t),p_2(t),\cdots,p_n(t))^T$ satisfied system (2.1), and $p(t)$ is a positive almost periodic solution of system (2.1).

Finally, we show that there is only one positive almost periodic solution of system (2.1). For any two positive almost periodic solution $p(t)=(p_1(t),p_2(t),\cdots,p_n(t))^T$ and $q(t)=(q_1(t),q_2(t),\cdots,q_n(t))^T$ of system (2.1), we claim that $p_i(t)=q_i(t)(i=1,2,\cdots,n)$ for all $t\in\mathbf{R}^+$. Otherwise there must be at least one positive number $\xi\in\mathbf{R}^+$ such that $p_i(\xi)\neq q_i(\xi)$ for a certain positive integer i, i.e., $\Phi=|p_i(\xi)-q_i(\xi)|>0$. So we can easily know that

$$\Phi=|\lim_{p\to+\infty}p_i(\xi+\delta_p)-\lim_{p\to+\infty}q_i(\xi+\delta_p)|=\lim_{p\to+\infty}|p_i(\xi+\delta_p)-q_i(\xi+\delta_p)|=\lim_{t\to+\infty}|p_i(t)-q_i(t)|>0,$$

which is a contradiction to (4.12). Thus $p_i(t)=q_i(t)$ $(i=1,2,\cdots,n)$ holds for $\forall t\in\mathbf{R}^+$. Therefore, system (1.1) admits a unique almost periodic solution which is globally attractive. This completes the proof of Theorem 5.1.

Theorem 5.2 Assume that (H1)-(H4) hold. Then systems (1.1) and (1.2) admit a unique positive almost periodic solution which is globally attractive.

Proof. From Lemma 2.5, we know that

$$(x_1(t),x_2(t),\cdots,x_n(t))^T=\big(\prod_{0<t_k<t}(1+h_{1k})y_1(t),\prod_{0<t_k<t}(1+h_{2k})y_2(t),\cdots,\prod_{0<t_k<t}(1+h_{nk})y_n(t)\big)^T$$

is a solution of systems (1.1) and (1.2). Since condition (H2) holds, similar to the proofs of Lemma 31 and Theorem 79 in [26], we can prove that $(x_1(t),x_2(t),\cdots,x_n(t))^T$ is almost periodic. Therefore, $(x_1(t),x_2(t),\cdots,x_n(t))^T$ is a unique globally attractive almost periodic solution of systems (1.1) and (1.2), because of the uniqueness and of global attractivity of $(y_1(t),y_2(t),\cdots,y_n(t))^T$ of system (2.1). This completes the proof.

Remark 5.1 If $\tau_i(t)=\tau_i$ and $\sigma_{ij}(t)=\sigma_{ij}(i,j=1,2,\cdots,n,j\neq i)$, where τ_i and σ_{ij} are nonnegative constants, the condition (H4) can be simplified. Therefore, we have the following result.

Corollary 5.1 Let $\tau_i(t)=\tau_i$ and $\sigma_{ij}(t)=\sigma_{ij}(i,j=1,2,\cdots,n,j\neq i)$, where τ_i and σ_{ij} are nonnegative constants. In addition to conditions (H1) and (H2) assume further that

$$\begin{aligned}\liminf_{t\to+\infty}\Big\{&B_i(t)-\frac{1}{(1+m_i)^2}\sum_{j=1,j\neq i}^{n}C_{ji}(t+\sigma_{ji})\big[1+\frac{M_j}{H_j^l}\int_{t+\sigma_{ji}}^{t+\tau_j+\sigma_{ji}}B_j(u)du\big]\\ &-\Big[a_i(t)+\frac{M_i}{H_i^l}B_i(t)+\sum_{j=1,j\neq i}^{n}\frac{M_jC_{ij}(t)}{H_j^l(1+m_j)}\Big]\int_t^{t+\tau_i}B_i(u)du-\frac{M_iB_i(t+\tau_i)}{H_i^l}\int_{t+\tau_i}^{t+2\tau_i}B_i(u)du\Big\}>0,\end{aligned}$$

$i=1,2,\cdots,n$. Then systems (1.1) and (1.2) admit a unique positive almost periodic solution which is globally attractive.

Remark 5.2 If $h_{ik}\equiv 0(i=1,2,\cdots,n,k\in\mathbf{R}^+)$, the condition (H4) can be simplified. Therefore, we have the following result.

Corollary 5.2 Let $h_{ik} \equiv 0 (i = 1, 2, \cdots, n, k \in \mathbf{R}^+)$. In addition to conditions (H1) and (H3) assume further that

$$\liminf_{t\to+\infty} \Bigg\{ b_i(t) - \frac{1}{(1+m_i)^2} \sum_{j=1,j\neq i}^{n} \frac{c_{ji}(\psi_{ji}^{-1}(t))}{\dot{\psi}_{ji}(\psi_{ji}^{-1}(t))} \Big[1 + M_j \int_{\psi_{ji}^{-1}(t)}^{\varphi_j^{-1}(\psi_{ji}^{-1}(t))} b_j(u)du \Big]$$
$$- \Big[a_i(t) + M_i b_i(t) + \sum_{j=1,j\neq i}^{n} \frac{M_j c_{ij}(t)}{1+m_j} \Big] \int_t^{\varphi_i^{-1}(t)} b_i(u)du - \frac{M_i b_i(\varphi_i^{-1}(t))}{\dot{\varphi}_i(\varphi_i^{-1}(t))} \int_{\varphi_i^{-1}(t)}^{\varphi_i^{-1}(\varphi_i^{-1}(t))} b_i(u)du \Bigg\} > 0,$$

$i = 1, 2, \cdots, n$. Then systems (1.1) and (1.2) admit a unique positive almost periodic solution which is globally attractive.

Remark 5.3 If $\sigma_{ij}(t) = 0 (i, j = 1, 2, \cdots, n, j \neq i)$, system (1.1) reduces to the system (1.1) in [1]. Some known results in [1] are improved and generalized.

6 An example

In this section, we give the following example to check the feasibility of our result.

Example Consider the following two species Lotka-Volterra mutualism system with time delays and impulsive effects:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big[0.3 - 0.05\sin(\sqrt{2}t) - (0.25 - 0.05\cos(\sqrt{3}t))x_1(t-0.01) + \dfrac{0.1x_2(t-0.02)}{1+x_2(t-0.02)}\Big], \\ \dot{x}_2(t) = x_2(t)\Big[0.3 - 0.05\cos(\sqrt{2}t) - (0.25 - 0.05\sin(\sqrt{3}t))x_1(t-0.02) + \dfrac{0.05x_1(t-0.01)}{1+x_1(t-0.01)}\Big], \\ x_1(t_k^+) = (1+h_k)x_1(t_k), \\ x_2(t_k^+) = (1+h_k)x_2(t_k), \quad k \in \mathbf{Z}^+. \end{cases} \tag{6.1}$$

where $\prod_{0<t_k<t} (1+h_k) \in [1, 1.1]$ is almost periodic, $t \in \mathbf{R}^+$.

A computation shows that

$$m_1 \approx 0.8262,\ M_1 \approx 2.1703,\ m_2 \approx 0.7371,\ M_2 \approx 2.2681,\ \tau = 0.02,$$

and moreover, we have

$$\liminf_{t\to+\infty} G_1(t) > 0.05 > 0, \quad \liminf_{t\to+\infty} G_2(t) > 0.05 > 0,$$

It is easy to see that the condition (H4) are satisfied. Hence, there exists a unique globally attractive almost periodic solution of system (6.1).

7 Concluding Remarks

In this paper, a multispecies Lotka-Volterra mutualism system with time-varying delays and impulsive effects is considered. Assume that the coefficients in system (1.1) are bounded non-negative almost periodic functions, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive. By comparative analysis, we find that when the coefficients in system (1.1) are almost periodic, the existence of a unique almost periodic solution of system (1.1) and (1.2) is determined by the global attractivity of system (1.1) and (1.2), which implies that there is no additional condition to add.

Furthermore, for the almost periodic multispecies Lotka-Volterra mutualism system with time-varying delays and feedback controls, we would like to mention here the question of whether the existence of a unique almost periodic solution is determined by the global attractivity of the system or not. It is, in fact, a very challenging problem, and we leave it for our future work.

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